Interference mitigation in GNSS signal acquisition through antenna arrays

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Introduction to GNSS

- Global Navigation Satellite Systems (GNSS) is the general concept used to identify those positioning systems that enable the determination of the user position and time by means of measured distances between the receiver and a set of visible satellites.

Figure: Position determination by GNSSs. Source: Kai Borre, Darius Plausinaitis, Reconfigurable GNSS Receivers, 2007.
GNSS signal structure

DS-CDMA transmitted signal for the $i$-th satellite:

$$s_{T,i}(t) = e_{I,i}(t) + je_{Q,i}(t),$$

where the subindex I and Q denote in-phase and quadrature component respectively, and are defined as:

$$e_{I,i}(t) = \sqrt{2P_{I,i}} \sum_{m_I = -\infty}^{\infty} b_{I,i}(m_I) \sum_{u_I = 1}^{N_{c_I}} \sum_{k_I = 1}^{L_{c_I}} c_{I,i}(k_I) g_I(t - m_I T_{b_I} - u_I T_{PRN_I} - k_I T_{c_I})$$

with the following definitions for the I component - analogous for the Q component:

- $P_{I,i}$ is the transmitted signal power.
- $b_{I,i}(m) \in \{-1, 1\}$ is the sequence of telemetry bits, with $T_{b_I}$ being the bit period.
- $c_{I,i}(k) \in \{-1, 1\}$ is the Pseudo Random Noise (PRN) spreading sequence, with the chip length of the codeword and the chip period defined as $L_{c_I}$ and $T_{c_I}$, respectively. $T_{PRN_I} = L_{c_I} T_{c_I}$ is the codeword period. The number of code epochs per telemetry bit is $N_{c_I}$.
- $g_I(t)$ is the energy-normalized chip shaping pulse.
Simplified receiver signal model

Considering only the LOSS contribution coming from $M_s$ satellites and the predominant propagation effects:

$$x_{RF}(t) = \Re \left\{ \sum_{i=1}^{M_s} \alpha_i(t)s_{T,i}(t - \tau_i(t))e^{j2\pi(f_c+f_{d,i}(t))t} \right\} + n(t), \quad (3)$$

where $f_c$ is the carrier frequency, $\theta_i = [\alpha_i(t), f_{d,i}(t), \tau_i(t)]$ are the complex amplitude, the Doppler frequency shift, and the time delay, respectively, which are the signal synchronization parameters for the $i$-th satellite signal and $n(t)$ is additive white Gaussian noise plus other undesired terms (such as multipath or interferences). The implicit channel model is defined as Wide-Sense Stationary Uncorrelated Scattering (WSSUS).
Figure: Simplified GNSS receiver high-level block diagram.
**Acquisition operation**

**Target:** provide a coarse estimation of the synchronization parameters vector $\theta_i$ for each received satellite:

$$\hat{\theta}_i = [\hat{\alpha}_i, \hat{f}_{d,i}, \hat{\tau}_i]$$

Figure: Acquisition test function output vs. time-delay and Doppler-shift of a noise-free and noisy Galileo-like MBOC(6,1,1/11) signal.
Acquisition parameters

Acquisition performance indicators

- **Sensitivity**
- \( TTFF_{cold} = T_{\text{warm-up}} + T_{\text{acq}} + T_{\text{track}} + T_{\text{NAV+GST}} + T_{\text{PVT}} \)

Performance degradation sources:

- Signal attenuation: fading and low level signal conditions.
- **Interferences**.
- Navigation message.
- Receiver and satellite movement.
- Receiver sample clock drift.
GNSS Interference sources

Among the **VHF and UHF spurious transmissions** (DVB-T, TACAN, and DME), the adjacent frequencies to GNSS links are populated as:

<table>
<thead>
<tr>
<th>Band [MHz]</th>
<th>Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1435-1530</td>
<td>Mobile (aeronautical telemetry)</td>
</tr>
<tr>
<td>1530-1545</td>
<td>Mobile-satellite (space-to-Earth)</td>
</tr>
<tr>
<td>1545-1549.5</td>
<td>Maritime mobile-satellite (space-to-Earth)</td>
</tr>
<tr>
<td><strong>1549.5-1558.5</strong></td>
<td>Aeronautical mobile-satellite (space-to-Earth)</td>
</tr>
<tr>
<td></td>
<td>Mobile-satellite (space-to-Earth)</td>
</tr>
<tr>
<td>1613.8-1626.5</td>
<td>Mobile-satellite (Earth-to-space)</td>
</tr>
<tr>
<td>1626.5-1645.5</td>
<td>Mobile-satellite (Earth-to-space)</td>
</tr>
<tr>
<td>1646.5-1651</td>
<td>Aeronautical mobile-satellite (Earth-to-space)</td>
</tr>
<tr>
<td>1651-1660</td>
<td>Mobile-satellite (Earth-to-space)</td>
</tr>
<tr>
<td>1668-1675</td>
<td>Meteorological aids (radiosonde)</td>
</tr>
<tr>
<td>1700-1710</td>
<td>Meteorological-satellite (space-to-Earth)</td>
</tr>
<tr>
<td>1710-1755</td>
<td>Mobile communications</td>
</tr>
</tbody>
</table>

Table: Sources and Services of image Interference for single-conversion GPS L1 / Galileo E1 front-end [Wil02].

The proposed deployment of Lightsquared’s 4G communication network could create a new interference situation specially when using the 1552.7 MHz band, as notified for the first time by the U.S. NTIA to U.S. FCC in January 2011.
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Problem definition

- New and upgraded GNSSs are being used to provide **safety-critical guidance**:
  - Modernized GPS and forthcoming Galileo will provide integrity monitoring mechanisms: RAIM, SBAS, GBAS, and Galileo embedded integrity concept.
  - Robust and highly **accurate positioning and time dissemination** service.
  - Used for the transport **safety of persons and goods**, such as in Civil aviation, Railroad transport, and Commercial maritime.

⇒ The **acquisition operation can become the bottleneck of GNSS receivers**:
  - Tracking initialization depends on acquisition results.
  - Acquisition has the lowest sensitivity of the whole receiver operation.
  - Strong **interferences and jamming signals** could increase the acquisition time or even **can make the acquisition fail**.
State-of-the-Art

Figure: Single-antenna receiver interference rejection.

Single-antenna solutions for acquisition interference mitigation

- **Time diversity**: pulse blanking and Automatic Gain Control (AGC).
- **Frequency diversity**: Continuous Wave Interference (CWI) notch filters.
- **Spatial diversity**: antenna radiation pattern.
Research opportunity

Figure: Array-based GNSS receiver.

- **Antenna-array key benefits:**
  - In addition to code, time and frequency diversities, arrays exploit **spatial diversity**.
  - **Interference and jamming** signals **rejection**.
  - Sensitivity improvement due to the array gain.

- **Acquisition with antenna arrays has not yet received much attention:**
  - Intense research activity on single antenna receivers.
  - Antenna arrays applied to tracking assuming the success of the acquisition.

⇒ **Existing antenna-array solutions suitable for acquisition:**
  - *Minimum Variance Distortionless Response (MVDR) beamformer:* requires an estimation of both the signal **DOA** and the array attitude.
  - *Spatial-domain grid search:* requires a calibrated array, but it increases the dimensions of the search grid.
  - *Blind null-steering beamformers:* **DOA not necessary**, but no array gain.
Research opportunity

- Lack of suitable array-based GNSS real-time receiver commercially available
  - Closed implementations with few technical information made public.

**Figure:** NovAtel GAJT commercial null-steering GNSS array.

- Scientific applications of GNSSs require flexible implementations
  - The **Software Defined Radio (SDR) receiver approach** is suitable to test new algorithms with low development costs.
  - Interest for a limited-market but highly demanding applications such as reference stations, geodesy and surveying, among other scientific goals.
Main objectives of the project

Exploit the spatial diversity in acquisition

- Improve the existing GNSS acquisition algorithms performance.
- **Improve the GNSS interference robustness** proposing **novel array-based acquisition algorithms** using the **statistical detection theory** framework.

Demonstrate the implementation feasibility

- Design and implementation of an array-based GNSS receiver platform
- **Proof-of-concept of array-based acquisition** algorithms working in **real-time**
  - Interference mitigation.
  - Performance measurements in **realistic scenarios**.
- Tightly coupled with GNSS software receivers.
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Antenna-array signal model

N-elements baseband array signal model for single satellite:

\[ X(t) = hd(t, f_d, \tau) + N(t), \]

where

- \( X(t) = [x_{bb}(t - (K - 1)T_s) \ldots x_{bb}(t)] \in \mathbb{C}^{N \times K} \) is referred as the *space-time data matrix*, where \( x_{bb}(t) = [x_1(t) \ldots x_N(t)]^T \) is defined as the baseband snapshot, and \( K \) is the number of captured snapshots. The acquisition time is defined as \( T_{acq} = KT_s \) where \( F_s = 1/T_s \) is the sampling frequency.

- \( h = [h_1 \ldots h_N]^T \in \mathbb{C}^{N \times 1} \) is the **non-structured channel model**, which includes both the channel and the array response.

- \( d(t, f_d, \tau) = [s_{T,i}(t - (K - 1)T_s - \tau_i)e^{j2\pi f_d,i(t-(K-1)T_s)}, \ldots, s_{T,i}(t - \tau_i)e^{j2\pi f_d,i}] \in \mathbb{C}^{1 \times K} \) is the received GNSS baseband signal for the \( i \)-th satellite.

- \( N(t) = [n(t - (K - 1)T_s) \ldots n(t)] \in \mathbb{C}^{N \times K} \) is a complex, circularly symmetric Gaussian vector process with a zero-mean, temporally white and **spatially colored** with an arbitrary (also unknown) **spatial covariance matrix** \( Q \in \mathbb{C}^{N \times N} \) that models both the noise and other non desirable terms such as interferences.
Detection theory

Binary hypothesis testing problem:

\[ \mathcal{H}_1 : X(t) = h d(t, f_d, \tau) + N(t) \quad (6) \]
\[ \mathcal{H}_0 : X(t) = N(t), \quad (7) \]

where \( \mathcal{H}_0 \) is the null hypothesis, when there is no signal present, and \( \mathcal{H}_1 \) is the alternative hypothesis, when the searched signal is present.
Neymann-Pearson (NP) detector

Likelihood Ratio Test (LRT):

\[
L(X) = \frac{p(X; Q, \theta_a, \mathcal{H}_1)}{p(X; Q, \mathcal{H}_0)} = \frac{\frac{1}{\pi^{NK} \det(Q)^K} \exp \left\{ - \text{Tr}\{Q^{-1}C\} \right\}}{\frac{1}{\pi^{NK} \det(Q)^K} \exp \left\{ - \text{Tr}\{Q^{-1}\hat{R}_{xx}\} \right\}} > \gamma, \tag{8}
\]

where the PDF of a complex multivariate Gaussian vector for \( K \) snapshots is used, and

\[
C = \hat{R}_{xx} - \hat{r}_{xd}h^H - h\hat{r}_{xd}^H + h\hat{R}_{dd}h^H, \tag{9}
\]

with

- \( \hat{R}_{xx} = \frac{1}{K}XX^H \) is the autocorrelation matrix estimation
- \( \hat{r}_{xd} = \frac{1}{K}Xd^H \) is the cross-correlation vector estimation
- \( \hat{R}_{dd} = \frac{1}{K}dd^H \) is the PRN code autocorrelation estimation
Generalized Likelihood Ratio Test (GLRT) detector

**GLRT criterion**: assume no *a priori* knowledge of the PDFs parameters and replace them by their respective Maximum Likelihood Estimators (MLE).

**GLRT expression**:

\[
L(X) = \frac{p(X; \hat{Q}_{\mathcal{H}_1}, \hat{\theta}_a, \mathcal{H}_1)}{p(X; \hat{Q}_{\mathcal{H}_0}, \mathcal{H}_0)} = \frac{1}{\pi^{NK} \det(\hat{Q}_{\mathcal{H}_1})^K} \exp \{ -\text{Tr}(\hat{Q}_{\mathcal{H}_1}^{-1}C) \} \frac{1}{\pi^{NK} \det(\hat{Q}_{\mathcal{H}_0})^K} \exp \{ -\text{Tr}(\hat{Q}_{\mathcal{H}_0}^{-1}\hat{R}_{xx}) \} > \gamma, \tag{10}
\]

where (\(\hat{\cdot}\)) is the MLE of the signal and noise parameters.

**Maximum Likelihood Estimators [FP06]**

\[
\hat{Q}_{\mathcal{H}_1} = C|_{h=\hat{h}, f_d=\hat{f}_d, \tau=\hat{\tau}} \tag{11}
\]

\[
\hat{Q}_{\mathcal{H}_0} = C|_{h=0} = \hat{R}_{xx} \tag{12}
\]

\[
\hat{h} = \hat{r}_{xd} \hat{R}_{dd}^{-1} |_{Q_{\mathcal{H}_1} = \hat{Q}_{\mathcal{H}_1}, f_d=\hat{f}_d, \tau=\hat{\tau}} \tag{13}
\]

\[
\hat{f}_d, \hat{\tau} = \arg \min_{f_d, \tau} \ln(\det(W(f_d, \tau))), \tag{14}
\]

where \(W(f_d, \tau) = \hat{R}_{xx} - \hat{r}_{xd}(f_d, \tau) \hat{R}_{dd}^{-1} \hat{r}_{xd}(f_d, \tau)^H = \hat{Q}_{\mathcal{H}_1} |_{h=\hat{h}, f_d, \tau} \).
GLRT statistics

Inserting (11) and (12) in (10), we obtain the test statistic function [Arr12]:

\[ T_{GL}(X) = \max_{f_d, \tau} \left\{ \hat{r}_{xd}^H(f_d, \tau) \hat{R}_{dd}^{-1} \hat{R}_{xx}^{-1} \hat{r}_{xd}(f_d, \tau) \right\} > \gamma, \]  \hspace{1cm} (15)

Assuming \( \hat{R}_{xx} \simeq R_{xx} \), \( T_{GL}(X) \) is distributed as:

\[ T_{GL}(X) \sim \begin{cases} \chi^2_{2N}(\delta_{T_{GL}; \mathcal{H}_1}), & \text{in } \mathcal{H}_1, \\ \chi^2_{2N}(\delta_{T_{GL}; \mathcal{H}_0} = 0), & \text{in } \mathcal{H}_0, \end{cases} \] \hspace{1cm} (16)

GLRT theoretical performance

\[ P_{fa}(\gamma) = \exp \left\{ \frac{-\gamma}{2\sigma^2_{T_{GL}}} \right\} \sum_{k=0}^{N-1} \frac{1}{k!} \left( \frac{\gamma}{2\sigma^2_{T_{GL}}} \right)^k, \] \hspace{1cm} (17)

\[ P_d(\gamma) = Q_N \left( \frac{\sqrt{\delta_{T_{GL}}(\hat{R}_{xx}, h)} \sqrt{\gamma}}{\sigma_{T_{GL}}}, \frac{\gamma}{\sigma_{T_{GL}}} \right), \] \hspace{1cm} (18)

where \( \sigma^2_{T_{GL}} = \frac{1}{2K} \) and \( Q_N \) is the generalized Marcum Q-function of order \( N \). The detector enjoys a Constant False Alarm Rate (CFAR) condition as \( P_{fa}(\gamma) \) does not depend on \( X \).
GLRT performance

The chi-square non-centrality parameter can be expressed as:

\[ \delta_{T_{GL}}(\hat{R}_{xx}, h) = \hat{R}_{dd}h^H\hat{R}_{xx}^{-1}h \sim h^H R_{xx}^{-1}h. \] (19)

Uniformly Most Powerful (UMP) condition

A UMP detector has the highest possible \( P_d \) for a given \( P_{fa} \).

Karlin-Rubin Theorem: Suppose that \( T(X) \) is a sufficient statistics for \( \theta \), and the family of PDFs associated to \( T(X) \) has a Monotone Likelihood Ratio (MLR). Then, the test that rejects \( H_0 \) if and only if \( T(X) > \gamma \) is a UMP test.

We check \( T_{GL}(X) \) against the MLR condition as

\[ \frac{p(T_{GL}(X); \delta_2)}{p(T_{GL}(X); \delta_1)} = \frac{p(\chi^2_{2N}(\delta_2))}{p(\chi^2_{2N}(\delta_1))}, \] (20)

where \( \delta_1 = \delta_{T_{GL}}(\hat{R}_{xx}, h_1) \) and \( \delta_2 = \delta_{T_{GL}}(\hat{R}_{xx}, h_2) \) are two possible values for the non-centrality parameter of the chi-square defined in (19). The ratio is monotone non-decreasing for all \( \delta_2 > \delta_1 \). The GLRT for GNSS array-based acquisition is a UMP test.
Simulation results: theoretical PDFs and histograms

Figure: Array-based GNSS acquisition concept.

(a) $T_{GL}(X)$

Figure: Histogram and theoretical PDF in both $\mathcal{H}_1$ and $\mathcal{H}_0$ acquisition hypotheses for Galileo E1 signal acquisition simulation (CN0=38 dB-Hz, no interferences).
GLRT interference rejection capability

Assume $M$ uncorrelated interferences impinging into the array through a channel matrix $H_i = [h_{i,1}, \ldots, h_{i,M}] \in \mathbb{C}^{N \times M}$ and an arbitrary basis function matrix $D_i = [d_{i,1}^T, \ldots, d_{i,M}^T]^T \in \mathbb{C}^{M \times K}$. Then, the noise term is modeled as:

$$N = H_i D_i + N_w,$$

where $N_w \sim \mathcal{CN}(0, \sigma^2 I)$. The inverse covariance matrix can be approximated as:

$$Q^{-1} \simeq \sigma^{-2} P_{H_i}^\perp,$$

where $P_{H_i}^\perp = (I - H_i (H_i^H H_i)^{-1} H_i^H)$ and $R_{D_i D_i} \simeq I$.

Assuming $\mathbb{E}\{\hat{R}_{xx}\} \simeq Q$, then

$$\delta_{T_{GL}}(\sigma^2, h, H_i) = \frac{h^H P_{H_i}^\perp h}{\sigma^2} = \frac{h^H h}{\sigma^2} - \frac{h P_{H_i} h^H}{\sigma^2},$$

where $P_{H_i} = H_i (H_i^H H_i)^{-1} H_i^H$. The SNR can be expressed as $\rho = \frac{h^H h}{\sigma^2} = N \frac{P_s}{P_n}$.
**GLRT interference rejection capability**

(a) $N = 2$

(b) $N = 3$

(c) $N = 4$

(d) $N = 8$

(e) $N = 12$

(f) $N = 24$

**Figure:** Evolution of $\delta_{TGL}(\rho, N, \hat{h}, \bar{h}_i)$ in the presence of an uncorrelated interference impinging into the array from $\text{Az} = 45^\circ$ and $\text{El} = 45^\circ$, with $\rho = 1$. 
Existing solutions: Acquisition after beamforming

Minimum Variance Distortionless Response (MVDR) GLRT test function

\[ T_{\text{MVDR}}(X) = \hat{r}_{xd}^H (R_{xx}^{-1} h_0)(R_{xx}^{-1} h_0)^H \hat{r}_{xd} \approx \frac{1}{\sigma^2} \frac{|\hat{r}_{xd}^H P_{H_i} h_0|^2}{\|h_0^H P_{H_i}\|^2}, \]  

(24)

where \( h_0 \in \mathbb{C}^{N \times 1} \) is the signal DOA steering vector.

Power Minimization Beamformer: signal DOA unavailable

Minimize the output power establishing a reference antenna

\[ h_0 = [1 \ 0 \ldots 0]. \]  

(25)
Simulation results: performance in ideal conditions

Figure: Galileo E1 acquisition $P_d$ vs. satellite CN0 in the presence of an in-band wideband interference for different algorithms ($IN0 = 85$ dB-Hz, $P_{fa} = 0.001$, $f_s = 6$ MSPS, $T_{acq} = 4$ ms). No pointing error in $T_{MVDR}(X)$. 

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Simulation results: performance with pointing error

Figure: Galileo E1 acquisition $P_d$ vs. satellite CN0 in the presence of an in-band wideband interference for different algorithms ($IN0 = 85$ dB-Hz, $P_{fa} = 0.001$, $f_s = 6$ MSPS, $T_{acq} = 4$ ms). 20° DOA pointing error in $T_{MVDR(X)}$. (array uncalibrated in acquisition)
Realistic conditions

- finite sample size
- signal quantization effects
- satellite signal synchronization errors

**Figure:** GNSS receiver block diagram, as simulated.
A Metric to measure the presence of errors in $\hat{R}_{xx}$

Geodesic distance: For any two points inside $S = \{ R \in \mathbb{C}^{N \times N} : R^H = R, R > 0 \}$, the geodesic distance is the length of the geodesic curve that connects them:

$$\text{dist}_g(R_1, R_2) = \| M \|_F,$$

(26)

where $\| M \|_F = \sqrt{\sum_i |\ln(\lambda_i)|^2}$ stands for the logarithmic version of the Frobenius norm, $M = \ln(\frac{1}{R_1^{\frac{1}{2}}} R_2 \frac{1}{R_1^{\frac{1}{2}}})$, and $\lambda_i$ represents the $i$-th eigenvalue of matrix $\frac{1}{R_1^{\frac{1}{2}}} R_2 \frac{1}{R_1^{\frac{1}{2}}}$. The geodesic distance is able to detect differences both in the eigenvectors and in the eigenvalues of $R_{xx}$ and $\hat{R}_{xx}$. 
Quantization effects in $\hat{R}_{xx}$

**Figure:** $\text{dist}_g(\hat{R}_{xx}, \hat{R}_{x_q x_q})$ vs. different quantization bits and an in-band CW interference impinging into the array with different $IN0$. 
Quantization effects in $\hat{R}_{xx}$

Receiver Operating Characteristic (ROC) curve

Figure: Effect of the $\text{dist}_g (R_{xx}, \hat{R}_{xx})$ in the acquisition performance of $T_{GL}(X)$, with an in-band CWI impinging into the array with $IN0 = 85$ dB-Hz.
Acquisition performance vs. ADC resolution

Figure: $P_d$ in an scenario with satellite $CN0 = 44$ dB-Hz, and an in-band CW interference impinging into the array with $IN0 = 70 - 136$ dB-Hz, for $P_{fa} = 0.001$ and $N_b = 2 - 8$ bits.
Front-end bandwidth effect in acquisition I

The pre-correlation SNR is defined as $\rho_{acq} = \frac{P'_s}{P'_n}$, where

$$P'_s = P_s \int_{-\frac{1}{2}}^{\frac{1}{2}} G_s(f) |H_{FE}(f)|^2 df,$$

$$P'_n = \frac{N_0}{2} B_{FE} \int_{-\frac{1}{2}}^{\frac{1}{2}} |H_{FE}(f)|^2 df,$$

with

- $N_0$ is the antenna noise density and $B_{FE}$ is the front-end pass-band bandwidth
- $G_s(f) = \frac{10}{11} G_{BOC(1,1)} + \frac{1}{11} G_{BOC(6,1)}$ is the PSD (for Galileo MBOC(6,1,1/11))
- $H_{FE}(f)$ is the DTFT of the front-end baseband-equivalent impulse response

Considering now $d' = d \ast h_{FE}$, using Wiener-Khinchine, we compute $R_{d'd'}$ as:

$$R_{d'd'}[\tau] = \int_{-\frac{1}{2}}^{\frac{1}{2}} G_s(f) H_{FE}(f) e^{j2\pi f \tau} df,$$

and therefore $P_s R_{d'd'}[0] = P_s \int_{-0.5}^{0.5} G_s(f) H_{FE}(f) df$. Now considering $H_{FE}(f)$ response of an ideal band-pass filter, then

$$P_s R_{d'd'}[0] = P_s \int_{-\frac{B_{FE}}{2fs}}^{\frac{B_{FE}}{2fs}} G_s(f) = P'_s.$$
Front-end bandwidth effect in acquisition II

Finally, by inserting $P'_s$ and $P'_n$ in the non-centrality parameter of the performance expressions we can write

$$\delta_{T_{GL}} = \frac{\mu_{R_{xd}}^2}{R_{xx}} \simeq \frac{P'_s}{P'_n} = \rho_{acq},$$

(31)

which implies that the maximization of $\rho_{acq}$ maximizes the acquisition performance.

![Graphs showing SNR vs. BW and ROC vs. BW](image)

**Figure:** Theoretical and simulated Galileo E1 MBOC(6,1,1/11) SNR and ROC curves vs. the baseband BW.
Front-end bandwidth effect in acquisition III

**Figure:** Autocorrelation of Galileo E1 MBOC(6,1,1/11) signal with the local replica versus $\tau$ for different baseband bandwidths.
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- Design and implementation of an array-based GNSS receiver platform
- **Proof-of-concept** of array-based acquisition algorithms working in **real-time**
  - **Interference mitigation**.
  - Performance measurements in **realistic scenarios**.
- Tightly coupled with GNSS software receivers.
Generic GNSS array-based receiver platform

Figure: Simplified system block diagram.

Figure: Simplified front-end for each antenna element.
## Front-end requirements overview

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Effect in Acquisition</th>
<th>Effect in Tracking</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑ Bandwidth (BW)</td>
<td>↓ sensitivity if BW exceeds the main lobe (SNR loss)</td>
<td>↑ ( \hat{\tau} ) precision (see RMSE CRB)</td>
</tr>
<tr>
<td>↑ Noise Figure (NF)</td>
<td>↓ sensitivity (( \downarrow ) CN0)</td>
<td>↓ ( \hat{\tau} ) precision (( \downarrow ) CN0)</td>
</tr>
<tr>
<td>↑ ADC active bits ((N_b))</td>
<td>↑ sensitivity (No CN0 ( \downarrow ) if ( N_b \geq 4 ))</td>
<td>↑ sensitivity (No CN0 ( \downarrow ) if ( N_b \geq 4 ))</td>
</tr>
<tr>
<td>↑ Linearity</td>
<td>↑ interference robustness</td>
<td>↑ interference robustness</td>
</tr>
</tbody>
</table>
Receiver performance bounds

**Figure:** GPS L1 C/A code tracking Cramér-Rao Bound (CRB) minimum RMSE vs. front-end NF, BW, and signal CN0 [Wei94].

<table>
<thead>
<tr>
<th>ADC resolution</th>
<th>CN0 degradation $f_s \geq 2B_s$</th>
<th>CN0 degradation $f_s = 2B_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 bit</td>
<td>1.96 dB</td>
<td>3.5 dB</td>
</tr>
<tr>
<td>2 bits</td>
<td>0.55 dB</td>
<td>1.2 dB</td>
</tr>
<tr>
<td>3 bits</td>
<td>0.16 dB</td>
<td>0.6 dB</td>
</tr>
<tr>
<td>4 bits or more</td>
<td>$\leq 0.16$</td>
<td>$\leq 0.6$ dB</td>
</tr>
</tbody>
</table>

**Table:** GPS L1 C/A CN0 degradation vs. ADC resolution assuming Gaussian noise [Par96].
Front-end specifications

- **Bandwidth, \( f_s \), and Noise Figure (NF)**: set by the desired GNSS performance.
- **Gain**: the thermal noise excites the ADC, thus:

\[
G_{FE} \geq P_{ADC} - P_n,
\]

\[
G_{FE} \geq 10\log \left( \frac{\left( \frac{V_{LSB}2^{N_{\text{bits}}-1}}{Z_{ADC}} \right)^2}{kT_aB_p} \right),
\]

where \( V_{LSB} \) is the Least Significant Bit voltage (RMS), \( N_{\text{bits}} \) is the number of active bits and \( P_n = kT_aB_p \) is the noise power where \( k = 1.3806 \times 10^{-23} \text{J/K} \) and \( T_a \) is the antenna sky temperature, and \( B_p \) is the passband bandwidth.

- **Linearity Parameters**: Compression Point (CP) and Third Order Interception Point (IP3) are set by the maximum admissible input power

\[
P_{\text{int max}} = P_{FSR} - G_{FE}.
\]

where \( P_{FSR} \) is the ADC full-scale input power. The front-end should behave linear for an input signal power \( P_{in} \leq P_{\text{int max}} \).

**ADC Particular case**: \( V_{FSR} = 2 \text{ Vpp}, Z_{ADC} = 50 \Omega, P_{FSR} = 19.1 \text{ dBm}, \) and \( N_b = 12 \).

**FE Specifications**: \( N_b \geq 4, P_{ADC} \geq -35 \text{ dBm}, P_n = -107.8 \text{ dBm}, G_{FE} \geq 72.8 \text{ dB} \)

\( P_{\text{int max}} = P_{FSR} - G_{FE} = -53.7 \text{ dBm}, P_{CP} \geq P_{\text{int max}} \) and \( P_{IP3} \geq (P_{\text{int max}} + 9.6) \).
Proposed front-end channel architecture and specifications

By applying the Friis formula we obtain the overall NF as:

\[
F_{FE} = F_{ant} + \frac{L_{cable} - 1}{G_{ant}} + \frac{F_{LNA1} - 1}{G_{ant}L_{cable}} + \frac{L_{RF,BPF} - 1}{G_{ant}G_{LNA1}L_{cable}} + \frac{F_{LNA2} - 1}{G_{ant}G_{LNA1}G_{LNA2}L_{cable}L_{RF,BPF}} + \frac{F_{MIX} - 1}{G_{ant}G_{LNA1}G_{LNA2}G_{MIX}L_{cable}L_{RF,BPF}L_{IF,BPF}} + \frac{L_{IF,BPF} - 1}{G_{ant}G_{LNA1}G_{LNA2}G_{MIX}L_{cable}L_{RF,BPF}L_{IF,BPF}},
\]

(34)

and

<table>
<thead>
<tr>
<th>Component</th>
<th>Compression point</th>
<th>Computed min. value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna LNA</td>
<td>( P_{CP,ant} \geq P_{CP,LNA1} - G_{ant} + L_{cable} )</td>
<td>−55.9 dBm</td>
</tr>
<tr>
<td>LNA1</td>
<td>( P_{CP,LNA1} \geq P_{CP,LNA2} - G_{LNA1} + L_{RF,BPF} )</td>
<td>−44.9 dBm</td>
</tr>
<tr>
<td>LNA2</td>
<td>( P_{CP,LNA2} \geq P_{CP,MIX} - G_{LNA2} )</td>
<td>−25.9 dBm</td>
</tr>
<tr>
<td>Mixer</td>
<td>( P_{CP,MIX} \geq P_{CP,VGA} - G_{MIX} + L_{IF,BPF} )</td>
<td>−10.9 dBm</td>
</tr>
<tr>
<td>VGA</td>
<td>( P_{CP,VGA} \geq P_{FSR} - G_{VGA} )</td>
<td>−15.9 dBm</td>
</tr>
</tbody>
</table>
Antenna array elements

**Figure:** Garmin GA27c active antenna.

**Figure:** Antenna array prototype and differences between radiation patterns of some antenna elements in elevation.
Local Oscillator distribution network

Figure: Eight ports Wilkinson PCB layout and prototype.

Figure: Measured output signal phase differences [°].
Multichannel PCB implementation

Figure: PCB design and prototype.

Figure: Front-end housing and setup on the antenna-array frame.
## Front-end channel design specifications and prototype performance summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulated Value</th>
<th>Measured Value</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF frequency</td>
<td>1575.42 MHz</td>
<td>1575.42 MHz</td>
<td>1575.42 MHz</td>
</tr>
<tr>
<td>IF frequency</td>
<td>70 MHz</td>
<td>69.9988 MHz</td>
<td>70 MHz (*)</td>
</tr>
<tr>
<td>Passband bandwidth ($B_p$)</td>
<td>17.8 MHz</td>
<td>17.5 MHz</td>
<td>$\geq 12$ MHz</td>
</tr>
<tr>
<td>Stopband bandwidth ($B_s$)</td>
<td>20 MHz</td>
<td>22 MHz</td>
<td>$\leq 20$ MHz (*)</td>
</tr>
<tr>
<td>$G_{FE}$</td>
<td>73.9 dB</td>
<td>$58 + G_{ant} = 73$ dB</td>
<td>$\geq 72.8$ dB</td>
</tr>
<tr>
<td>$NF_{FE}$</td>
<td>2.178 dB</td>
<td>2.18 dB</td>
<td>$\leq 4$ dB</td>
</tr>
<tr>
<td>$P_{CP,FE}$</td>
<td>$-$</td>
<td>$-65.3$ dB</td>
<td>$&gt;-44.9$ dBm (*)</td>
</tr>
<tr>
<td>$P_{IP3,FE}$</td>
<td>$-$</td>
<td>$-65.3 + 9.6 = -54.7$ dBm</td>
<td>$&gt;-35.3$ dBm (*)</td>
</tr>
<tr>
<td>Image rejection</td>
<td>60.2 dB</td>
<td>57 dB</td>
<td>$\geq 40$ dB</td>
</tr>
<tr>
<td>Phase noise (10 kHz)</td>
<td>$-75.65$ dBc</td>
<td>$-82$ dBc</td>
<td>$-$</td>
</tr>
</tbody>
</table>

**Table:** Simulated and measured front-end performance compared to design specifications.

- **Out-of-specs linearity:** Considering an in-band interference signal present at the antenna terminals with $P_{int} = P_{CP,FE}$, the front-end linearity is sufficient to test the algorithms interference mitigation capability for $IN0 \leq 113$ dB-Hz considering $T_a = 100$ K.
FPGA Digital processing detailed block diagram
Autocorrelation matrix estimation implementation

\[
\hat{R}_{xx} = \frac{1}{K} \mathbf{X} \mathbf{X}^H = \\
= \frac{1}{K} \begin{bmatrix}
\sum_{k=1}^{K} x(k) x_1^*(k), & \ldots, & \sum_{k=1}^{K} x(k) x_N^*(k) \\
\sum_{k=1}^{K} x_1(k) x_1^*(k) & \ldots & \sum_{k=1}^{K} x_1(k) x_N^*(k) \\
\vdots & \ddots & \vdots \\
\sum_{k=1}^{K} x_N(k) x_1^*(k) & \ldots & \sum_{k=1}^{K} x_N(k) x_N^*(k)
\end{bmatrix} = \hat{R}_{xx}
\]

(35)
Real-time logic for $\hat{R}_{xx}$.
Cross-correlation vector estimation implementation

\[
\hat{r}_{xd}(\tilde{f}_d, \tilde{\tau}) = \frac{1}{K} X_d(\tilde{f}_d, \tilde{\tau})^H = \\
= \left[ \sum_{k=1}^{K} x_1(k)e^{-j2\pi \tilde{f}_d d^*(k, \tilde{\tau})}, \ldots, \sum_{k=1}^{K} x_N(k)e^{-j2\pi \tilde{f}_d d^*(k, \tilde{\tau})} \right].
\] (36)
Real-time logic for $\hat{r}_{xd}$. 
High-level operations: FPGA embedded software-defined processor
Calibration process

1. **Generate a reference carrier** $f_{\text{cal}} = f_c + f_{\text{ref}}$, capture $X$, and **estimate the amplitudes**
   
   vector $a_{\text{cal}} = [\frac{1}{\sqrt{\hat{P}_1}}, \ldots, \frac{1}{\sqrt{\hat{P}_N}}] \in \mathbb{R}^{1 \times N}$, where $\hat{P}_i = \frac{1}{K} \sum_{0}^{K-1} x_i x_i^*$

2. **Estimate the phase shifts** between the reference channel and rest of the channels using the ML estimator and compute $\phi_{\text{cal}} = \left[ e^{-j2\pi \frac{\Delta k_i f_{\text{ref}}}{f_s}}, \ldots, e^{-j2\pi \frac{\Delta k_i f_{\text{ref}}}{f_s}} \right] \in \mathbb{C}^{1 \times N}$.

3. **Compute** $G_{\text{cal}} = \text{diag}(a_{\text{cal}}) \text{diag}(\phi_{\text{cal}})$ and **apply it to the snapshots** as $X_{\text{cal}} = G_{\text{cal}} X$.

---

**Figure:** Calibration setup in an anechoic chamber.

**Figure:** Uncalibrated and calibrated snapshots plots for $f_{\text{ref}} = 10$ kHz.
A spectral Multiple Signal Classification (MUSIC) algorithm was used to estimate a test signal DOA and verify the array calibration:

\[
(\hat{\theta}, \hat{\psi}) = \arg \max_{\theta, \psi} \frac{1}{v(\theta, \psi)H (I - uu^H)v(\theta, \psi)},
\]

where \(v(\theta, \psi) \in \mathbb{C}^{N \times 1}\) is the signal steering vector, \(u \in \mathbb{C}^{N \times 1}\) is the eigenvector associated with the most powerful eigenvalue of \(\hat{R}_{xx}\), and \(I \in \mathbb{R}^{N \times N}\) is the identity.

**Figure:** DOA elevation estimation pointing error for \(\theta = 0^\circ\) and \(\theta = 180^\circ\) (The array was calibrated in the broadside direction \(\psi = 90^\circ\)).
Calibration validation II

**Figure:** MUSIC signal DOA estimation for $\theta = 0^\circ, \psi = 45^\circ$.

**Figure:** MUSIC signal DOA estimation for $\theta = 180^\circ, \psi = 45^\circ$.

**Figure:** MUSIC signal DOA estimation for $\theta = 0^\circ, \psi = 75^\circ$.

**Figure:** MUSIC signal DOA estimation for $\theta = 180^\circ, \psi = 75^\circ$. 
Interference scenario setup

- A GPS L1 C/A satellite signal is transmitted using a horn antenna, with $\theta = 0^\circ$ and $\psi = 90^\circ$, simulating a realistic situation were a high elevation satellite is received with $C/N_0 = 44$ dB-Hz.

- An interference signal is transmitted using an auxiliary monopole antenna with an approximated DOA of $\theta = 45^\circ$ and $\psi = 45^\circ$, which simulates a moderate elevation jammer or a communication signal coming from nearby communication tower.

- The interference signal and the satellite signal were generated by two Agilent E4438C generators. The latter was equipped with GPS Personality.

- The test functions were executed using $T_{\text{acq}} = 1$ ms of captured snapshots.
Interference scenario setup
Implemented acquisition functions

- **GLRT colored**
  \[
  T_1(X) = \max_{f_d, \tau} \left\{ \hat{r}^H_{xd}(f_d, \tau) \hat{R}_{dd}^{-1} \hat{R}_{xx}^{-1} \hat{r}_{xd}(f_d, \tau) \right\} > \gamma. \tag{38}
  \]

- **GLRT white**
  \[
  T_2(X) = \max_{f_d, \tau} \left\{ \frac{\hat{r}_{xd}(f_d, \tau)^H \hat{r}_{xd}(f_d, \tau)}{\text{Tr}(\hat{R}_{xx})} \right\} > \gamma. \tag{39}
  \]

- **Single antenna MF**
  \[
  T_3(X, i) = \max_{f_d, \tau} \left\{ \sum_{t=0}^{K-1} \left\| x_i(t) e^{-j2\pi f_d d^* (t, \tau)} \right\|^2 \right\} > \gamma, \tag{40}
  \]
  where \(i\) is the selected antenna element.
Acquisition results for CW interference

A **Continuous Wave (CW) jammer** impinges into the array with $f_{\text{int}} = 1575.43$ MHz and $J/N_0 = 133$ dB-Hz.

(a) IF spectrum.

(b) $T_{\text{Single}}(x(t,\tilde{f}_d, \tilde{\tau}))$.

(c) $T_{\text{GL White}}(X(t,\tilde{f}_d, \tilde{\tau}))$.

(d) $T_{\text{GL Colored}}(X(t,\tilde{f}_d, \tilde{\tau}))$. 
A possible **Lightsquared interference** was simulated using a Long Term Evolution (LTE) downlink signal with 5 MHz of bandwidth, impinging into the array with $f_{\text{int}} = 1575.42$ MHz and $J/N_0 = 133$ dB-Hz.

(e) IF spectrum.

(f) $T_{\text{Single}}(t, \tilde{f}_d, \tilde{\tau})$.

(g) $T_{\text{GL White}}(t, \tilde{f}_d, \tilde{\tau})$.

(h) $T_{\text{GL Colored}}(t, \tilde{f}_d, \tilde{\tau})$. 

---

**Acquisition results for 4G/LTE**
The deflection coefficient is defined as

\[ d^2 = \frac{\left( \mathbb{E}\{T(X; \mathcal{H}_1)\} - \mathbb{E}\{T(X; \mathcal{H}_0)\} \right)^2}{\text{var}\{T(X; \mathcal{H}_0)\}}. \] (41)

In the measurements, the expectations operators and the variance operator were substituted by its sample mean and sample variance respectively.

(i) CW interference scenario.

(j) LTE interference scenario.
Introduction

Motivation and Objectives

Array-based acquisition theory

Array platform implementation

Conclusions
Theoretical analysis

Done:

✓ **GLRT extension to the GNSS array signal model** assuming an **arbitrary and unknown covariance noise matrix**. The obtained detector is able to **mitigate temporally uncorrelated interferences** even if the array is **unstructured** and moderately **uncalibrated**.

✓ The performance and the interference rejection capability were modeled and compared to their theoretical bound. The detector was proven to be a **UMP test detector and has CFAR properties**.

✓ The proposed acquisition algorithm was compared to conventional acquisition after the MVDR beamformer. The **MVDR is severely affected by moderate pointing errors and/or array miss-calibration**, while the GLRT detector remains insensible to this situation.

✓ **Analysis of realistic conditions** to obtain preliminary **hardware requirements** for an implementation of the algorithm.

✓ The effect of the signal bandwidth in the acquisition has been modeled: **an increase of the bandwidth beyond the main spectrum lobe reduces the SNR, and, consequently, reduces also the performance of the acquisition algorithms.**
Theoretical analysis

To Do:

- Explore the application of the **Bayesian approach** to the acquisition problem; it might be possible to estimate a prior PDF for the synchronization **parameters** and integrate them in the GLRT detector.
- Integration of **inertial sensors** in the receiver at acquisition stage.
- Ultimately, an **hybridization between array-based acquisition and tracking** might be possible, since both algorithms are related in the sense that acquisition contains prior information suitable for tracking initialization and vice-versa.
Design and implementation of a real-time array-based GNSS receiver platform

Done:

✓ A novel real-time array-based GNSS receiver platform was developed:

✓ A complete signal reception chain, including the antenna array and the multichannel RF front-end for the GPS L1/ Galileo E1 was designed, implemented and tested.

✓ The FPGA digital processing section of the platform, such as the array signal statistics extraction modules were also implemented.

✓ Coupled with a software defined receiver.

✓ Design trade-offs and implementation complexities were carefully analyzed and taken into account.

✓ Proof-of-concept prototype: the array-based acquisition algorithm introduced in this Dissertation was implemented and tested under realistic conditions.

✓ Harsh in-band interference scenarios were tested, including narrow/wide band interferers and communication signals. The performance results were aligned with the theoretical and simulated ones, thus, completing the algorithm validation.
To Do:

- The platform can be enhanced by **integrating an IMU**.
- Design and implementation of **dual-band antenna elements patches** to cover both the GPS L1 / Galileo E1 and the GPS L5 / Galileo E5a links, which provide a complete SoL signals coverage of two of the most evolved GNSS.
- Explore the **integration of array elements and front-end calibration procedures**.
- Explore the application of **open source soft processors** such as the Aeroflex Gaisler LEON3 to replace the Microblaze implementation. Ultimately, it should be possible to **integrate the GNU Radio framework and the GNSS-SDR software receiver in the soft processor**.
Interference mitigation in GNSS signal acquisition through antenna arrays

Dr. Javier Arribas
(jarribas@cttc.es)

Interference Management for Tomorrow’s Wireless Networks
Newcom# Summer School Sophia Antipolis - May 28-31, 2013

Thank you for your attention!
Existing solutions: Acquisition after beamforming

Beamformer output signal model and GLRT test function:

\[ y = w^H X, \]  
\[ T_{DBF}(X) = \max_{f_d, \tau} \frac{\hat{R}^H_{yd}(f_d, \tau)\hat{R}_{yd}(f_d, \tau)}{\hat{R}_{yy}} > \gamma, \]  
where \( w \in \mathbb{C}^{N \times 1} \) is the beamformer weights vector, and

\[ \hat{R}_{yd} = \frac{1}{K} yd^H = w^H \frac{1}{K} Xd^H = w^H \hat{R}_{xd} \]  
\[ \hat{R}_{yy} = \frac{1}{K} yy^H = w^H \hat{R}_{xx} w. \]
Existing solutions: Acquisition after beamforming

Test function distribution

\[ T_{\text{DBF}}(X) \sim \begin{cases} \chi_2^2(\delta_{\text{DBF}}; \mathcal{H}_1), & \text{in } \mathcal{H}_1, \\ \chi_2^2(\delta_{\text{DBF}}; \mathcal{H}_0 = 0), & \text{in } \mathcal{H}_0, \end{cases} \tag{46} \]

where \( \sigma_{\text{DBF}}^2 = \frac{1}{2K} \) and

\[ \delta_{\text{DBF}}; \mathcal{H}_1 = \frac{\|\hat{\mu}_y\|^2}{\hat{R}_{yy}} = \frac{w^H R_{dd} h h^H R_{dd}^* w}{w^H R_{xx} w}. \tag{47} \]

Performance expressions

\[ P_d(\gamma) = Q_1 \left( \frac{\sqrt{\delta_{\text{DBF}}; \mathcal{H}_1}}{\sigma_{\text{DBF}}}, \frac{\sqrt{\gamma}}{\sigma_{\text{DBF}}} \right), \tag{48} \]

\[ P_{fa}(\gamma) = \exp \left\{ -\frac{\gamma}{2\sigma_{\text{DBF}}^2} \right\}, \tag{49} \]

where \( Q_1 \) is the generalized Marcum Q-function of order 1. The acquisition test statistic is CFAR.
Existing solutions: MVDR beamformer

Minimum Variance Distortionless Response (MVDR) optimum weights

\[
\begin{align*}
\min_w & \quad w^H R_{xx} w \\
\text{s.t} & \quad w^H h_0 = 1,
\end{align*}
\]

(50)

where \( h_0 \in \mathbb{C}^{N \times 1} \) is the DOA steering vector. Then,

\[
 w_{\text{opt}} = \frac{R_{xx}^{-1} h_0}{h_0^H R_{xx}^{-1} h_0}.
\]

(51)

MVDR GLRT test function

\[
T_{\text{MVDR}}(X) = \frac{\hat{r}_{xd}^H (R_{xx}^{-1} h_0)(R_{xx}^{-1} h_0)^H \hat{r}_{xd}}{h_0^H R_{xx}^{-1} h_0} \sim \frac{1}{\sigma^2} \frac{|\hat{r}_{xd}^H P_{H_i} h_0|^2}{\|h_0^H P_{H_i}\|^2}.
\]

(52)
Existing solutions: Power minimization beamformer

**Design condition:** the DOA is unavailable. The beamformer minimizes the output power while not weighting a reference antenna ($h_0 = [10 \ldots 0]$).

**Performance in absence of interferences** ($P_{\perp_{H_i}} = I$)

$$TPWR(X) = \sigma^{-2}\|r_{xd}^H h_0\|^2 = T_{GL}(x_1, f_d, \tau).$$ \hspace{1cm} (53)

*Figure:* $TPWR(X)$ histogram and theoretical PDF in both $H_1$ and $H_0$ acquisition hypotheses for Galileo E1 signal acquisition simulation.


